2015 Che2410 – Homework Assignment #3
Due on Oct. 23rd before midnight

1. (30 pts) Consider the heat (diffusion) equation:

$$\frac{\partial T}{\partial t} = \mathrm{K}\frac{\partial^2 T}{\partial x^2}, \qquad K = 5.0$$

$$T(t,0) = 50\sin(t) + 300, \qquad T(t,10) = 300, \qquad 0 \le t \le 4\pi$$
$$T(0,x) = 300, \qquad 0 \le x \le 10$$

We are modelling the temperature profile of a long and narrow reactor, where the temperature at one end is room temperature, and at the other end the temperature is fluctuating ± 50 degrees around room temperature.

(a) Turn this differential equation into a difference equation by using the Forward Euler method (i.e., forward differencing in time, central differencing in space).

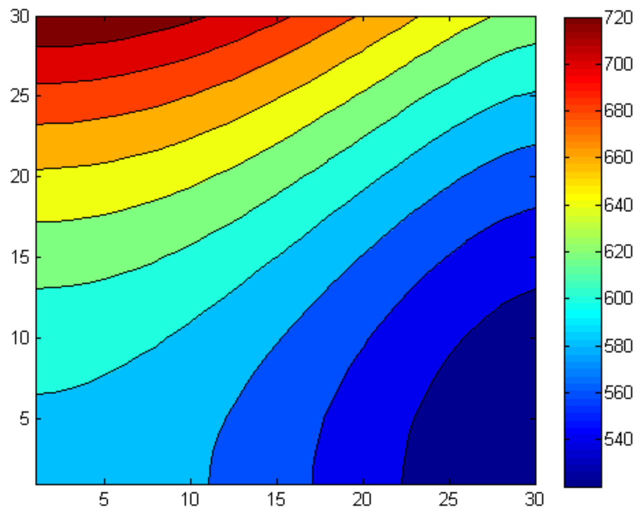
(b) Solve the difference equation using the scheme in (a). Use an extra grid point on each end of the x-domain to handle the boundaries (i.e., do not use periodic boundary conditions). Plot the temperature profile of the entire reactor at $t = 4\pi$. Also, show your results for the entire time-domain and x-domain by representing the T variable by color: create a 2D plot with x on the horizontal axis, t on the vertical axis, and represent T by color (where blue is coldest and red is warmest). See example below.

(c) Turn this differential equation into a difference equation by using the Crank-Nicolson scheme.

(d) Solve the difference equation above with the same boundary conditions as in (b). Also report your results using the same plots as in (b).

(e) How do your results compare using the two different methods?

(f) For either method, plot your results also for $K = 50$ and $K = 500$. What changes, what does not change, and why?



Example of a 2D temperature plot