

2015 Che2410 – Homework Assignment #1

Due on Sept. 22nd at 4:30pm

1. Classify the following differential equations. State whether they are linear/nonlinear, ordinary/partial, homogeneous/inhomogeneous and state their order. If applicable, state whether the equation is elliptic, hyperbolic, or parabolic, and under what conditions (if any).

a) $f''' + f' = 0$

b) $\frac{dy}{dx} + x^2 \frac{d^2y}{dx^2} = \frac{1}{x}$

c) $f_t f_x = 1$

d) $y(x) + y'(x) + xy''(x) = 0$

e) $(x^2 + 4)dy = (2x - 8xy)dx$

f) $2f_{xx} + 4f_{xy} - f_{yy} + f_x = 0$

g) $x \frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 y}{\partial t^2} + 2 \frac{\partial^2 y}{\partial x \partial t} = 1$

h) $\nabla^2 \phi = 0$

2. Solve the following 1st order differential equations:

a) $x^2 \frac{df}{dx} + 4f = 2$

b) $(x^2 + 4) \frac{dy}{dx} + 8xy = 2x$

3. Derive the formula for the 6th order central difference operator. Determine the leading order term in the truncation error.

4. Derive the 4th order approximation to the second derivative. Determine the leading order term in the truncation error.

5. Derive the formulas for the 2nd and 3rd order forward differencing.

6. Consider the equation:

$$f_t = f_x$$

Show that the MacCormack scheme is algebraically equivalent to the Lax-Wendroff scheme.